

# Radiative spacetimes

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**Abstract.** The question of existence of general, asymptotically flat radiative spacetimes and examples of explicit classes of radiative solutions of Einstein's field equations are discussed in the light of some new developments. The examples are cylindrical waves, Robinson-Trautman and type N spacetimes and especially boost-rotation symmetric spacetimes representing uniformly accelerated particles or black holes.

## 1 Introduction

In physical theories on a fixed background spacetime, as in Newtonian theory or special relativity, it is not difficult to formulate asymptotic fall-off conditions on fields of spatially bounded systems. For example, the gravitational potential due to a Newtonian star is usually required to decay to zero at infinity of Euclidean space, with the decay rate being compatible with Laplace's equation. In general relativity no a priori given background space exists. The metric itself is both a dynamical field and a quantity which determines distances. One expects that in a suitable coordinate system far away from a system of bodies the metric should have a form  $g_{\mu\nu} = \eta_{\mu\nu} + \text{small quantities}$ , where  $\eta_{\mu\nu}$  is Minkowski metric. What, however, does it mean "far away", what is "infinity"? Can one formulate suitable boundary conditions in a coordinate-free manner? What is the decay of a *radiative* gravitational field?

After several important contributions to the gravitational radiation theory in the late 1950's and early 1960's by Pirani, Bondi, Robinson, Trautman and others, a landmark paper by Bondi et al [1] appeared in which radiative properties of isolated (spatially bounded) axisymmetric systems were studied along outgoing null hypersurfaces  $u = \text{constant}$ , with  $u$  representing a retarded time function. An ansatz was made that the metric along  $u = \text{constant}$  can be expanded in inverse powers of  $r$ ,

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}(\theta)r^{-1} + f_{\mu\nu}(\theta, \varphi)r^{-2} + \dots, \quad (1)$$

where  $r$  denotes a suitable parameter along null generators (parametrized by coordinates  $\theta, \varphi$ ) on the hypersurfaces  $u = \text{constant}$ . Under the assumption (1) Einstein's vacuum equations were shown to determine uniquely formal power series solution of the form (1), provided that a free "news function"

$c(u, \theta)$  is specified. The news function contains all information about radiation at infinity ("r =  $\infty$ "). It enters the fundamental "Bondi mass-loss formula" for the total mass  $M(u)$  of an isolated system at retarded time  $u$ . Field equations imply that  $M(u)$  is a monotonically decreasing function of  $u$  if  $\partial_u c \neq 0$ . A natural interpretation is that gravitational waves carry away positive energy from the system and thus decrease its mass. In the work of Bondi et al [1] as well as in the important generalizations by Sachs, Newman and Penrose, the decay of radiative fields was studied in preferred coordinate systems.

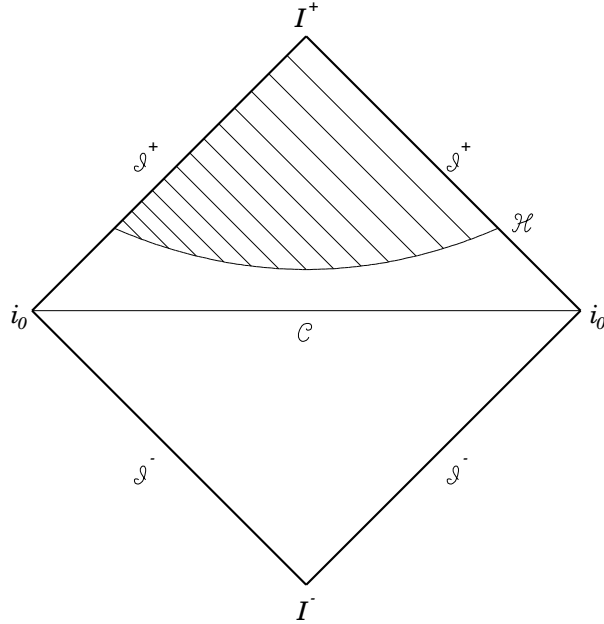
In 1963 Penrose [2] formulated a beautiful *geometrical* framework for description of the "radiation zone" in general relativity in terms of conformal infinity.

Penrose's definition of asymptotically flat radiative spacetimes avoids such problems as "distances" or "suitable coordinates", and incorporates a clear definition of what is infinity. It is inspired by the work on radiation theory mentioned above, and by the properties of conformal infinity in Minkowski spacetime. In contrast to an Euclidean space, in Minkowski spacetime one can go to infinity in various directions: moving along timelike geodesics we come to the future (or past) timelike infinity  $I^+$  (or  $I^-$ ); along null geodesics (cf. Eq. (1)) we reach the future (past) null infinity  $\mathcal{J}^+$  ( $\mathcal{J}^-$ ); spacelike geodesics lead to spatial infinity  $i_0$ . Minkowski spacetime can be compactified and mapped into a finite region by an appropriate conformal transformation. Thus one obtains the well-known *Penrose diagram* in which the three types of infinities are mapped into the boundaries of the compactified spacetime - see Fig. 1.

It is generally accepted that Penrose's definition forms the only rigorous, geometrical basis for the discussion of gravitational radiation from isolated systems. It enables us to use techniques of local geometry "at infinity" and to define covariantly such fundamental quantities as the total (Bondi) mass of an isolated system.

## 2 Asymptotically flat radiative spacetimes: existence

Despite its rigour and elegance, Penrose's *definition* might turn to be of a limited importance if no interesting radiative spacetimes *exist* which satisfy the definition. In Section 5 we shall describe special exact radiative spacetimes which represent "uniformly accelerated" sources in general relativity and admit  $\mathcal{J}$  as required; however, at least four points on  $\mathcal{J}$  - those in which worldlines of the sources start and end - are singular. There are no other *explicit* exact radiative solutions describing finite sources available at present and this situation will probably not change soon. Nevertheless, thanks to the work of Friedrich [3] and Christodoulou and Klainerman [4] we know that globally non-singular (including *all*  $\mathcal{J}$ ) asymptotically flat exact solutions of Einstein's equations really exist.



**Fig. 1.** The Penrose conformal diagram of an asymptotically flat spacetime. The Cauchy hypersurface and the hyperboloidal hypersurfaces are indicated.

The key idea of Friedrich is in realizing that Penrose's treatment of infinity not only permits to use the methods of local differential geometry at  $\mathcal{I}$  but also to analyze global existence problems of solutions of Einstein's equations in physical spacetime  $M$  by solving initial value problems in unphysical spacetime  $\tilde{M}$ .

By using this approach Friedrich established that formal Bondi-type expansions (1) converge locally at  $\mathcal{I}^+$ . He succeeded to show that one can formulate the "hyperboloidal initial value problem" for Einstein's vacuum equations in which initial data are given on a hyperboloidal spacelike hypersurface  $\mathcal{H}$  which intersects  $\mathcal{I}$  (see Fig. 1). It can then be proven that hyperboloidal initial data, which are sufficiently close to Minkowskian hyperboloidal data (i.e. to the metric induced on the hypersurface  $\mathcal{H}$  in Minkowski spacetime by the standard Minkowski metric) evolve to a vacuum spacetime which is smooth on  $\mathcal{I}^+$  and  $I^+$  as required by Penrose's definition.

Despite the deep and complicated work by Friedrich, one still did not have what one really would wish. Let us first note that initial data "*sufficiently close*" to Minkowskian data in Friedrich's result we mentioned above, do not mean any approximation - "smallness" of data is understood in a

functional sense in an appropriate Sobolev space. One cannot hope to prove global existence of smooth general solutions developed from general, arbitrarily *strong* data. Vacuum data representing strong gravitational waves could, due to nonlinearities, lead to creation of black holes with singularities inside. Nevertheless, ultimately one would like to have also results on the evolution of strong data.

More importantly, however, we would like to have initial data given on a *standard* spacelike Cauchy hypersurface which does not intersect  $\mathcal{J}$  but "ends" at spatial infinity (cf. hypersurface  $\mathcal{C}$  in Fig. 1), rather than data given on a hyperboloidal initial hypersurface. It could well happen that a spacetime evolved from data on  $\mathcal{H}$  is smooth "above"  $\mathcal{H}$  (in the shaded region in Fig. 1), however, it does not satisfy Penrose's requirements of asymptotic flatness "below"  $\mathcal{H}$ .

A remarkable progress in proving rigorously the existence of general, asymptotically flat radiative spacetimes was achieved by Christodoulou and Klainerman. Their treatise [4] contains the first really global general existence statement for full, nonlinear Einstein's vacuum equations with vanishing cosmological constant: Any smooth asymptotically flat initial data set (determined by the first and the second fundamental form on a Cauchy hypersurface) which is "near flat (Minkowski) data" leads to a unique, smooth and geodesic complete development solution of Einstein's vacuum equations. This solution is "globally asymptotically flat" in the sense that the curvature tensor decays to zero at infinity in all directions. The Christodoulou-Klainerman theorem involves a "global smallness assumption" which requires appropriate (integral) norms of curvature tensor of initial data integrated over the initial Cauchy hypersurface to be small.

The theorem demonstrates the existence of singularity-free, asymptotically flat radiative vacuum spacetimes. In hydrodynamics, even for arbitrary small initial data (decaying at infinity), an analogous theorem is not true since shocks arise. In general relativity the effect of nonlinear terms, which could have led to formation of singularities, is excluded owing, apparently, to the covariance and algebraic properties of the Einstein vacuum equations. That singularities could have had well developed is evident: the collision of arbitrarily weak, vacuum plane gravitational waves leads (due to nonlinearities) to the formation of a singularity. This, of course, does not contradict the Christodoulou-Klainerman result because initial data for plane waves are not asymptotically flat.

The work of Friedrich, Christodoulou, Klainerman and others demonstrates rigorously that the general picture of null infinity is compatible with the vacuum Einstein field equations. However, important open questions remain. In classical papers by Bondi et al [1], Newman and Penrose and others, the decay of the curvature (characterized by the Weyl tensor) along outgoing null geodesics at infinity exhibits "the peeling-off" property: the fall-off of various components of the Weyl tensor is related to their Petrov algebraic type.

To be more specific, certain complex linear combinations of the Weyl tensor in the orthonormal frame,  $\Psi_k$  ( $k = 0, 1, 2, 3, 4$ ), behave as  $\Psi_k = O(r^{k-5})$  as  $r \rightarrow \infty$ . (In particular,  $\Psi_4 \sim r^{-1}$  has the same algebraic structure as the Weyl tensor of a plane wave – radiative field of a bounded system resembles asymptotically that of a plane wave.) This decay of the curvature can be shown to follow from a sufficient differentiability (smoothness) of the conformally rescaled (unphysical) metric  $\tilde{g}$ . A sufficient smoothness of null infinity is thus commonly assumed. The results of Christodoulou and Klainerman, however, show a weaker peeling. They were only able to prove that the asymptotically flat vacuum initial data lead to  $\Psi_0 \sim r^{-\frac{7}{2}}$  (*not*  $\sim r^{-5}$ ) at null infinity.

An increasing evidence against the proposal of a smooth  $\mathcal{I}$  has led Chruściel et al [5] to introduce the concept of a *polyhomogeneous*  $\mathcal{I}$ . The metric is called polyhomogeneous if at large  $r$  it admits an expansion in terms of  $r^{-j} \log^i r$  rather than  $r^{-j}$  (as it has been assumed in the works of Bondi and others - cf. Eq. (1)). The hypothesis of polyhomogeneity of  $\mathcal{I}$  has been shown to be formally consistent with Einstein's vacuum equations. Under appropriate assumptions on the asymptotic form of the polyhomogeneous metric, one can demonstrate that the Bondi mass-loss law can be formulated, and the peeling-off property of the curvature holds, with the first two terms identical to the standard peeling, the third term being  $\sim r^{-3} \log r$ . At null infinity the conformally rescaled (unphysical) metric is not smooth. Although a substantial progress in understanding the existence of radiative solutions of vacuum field equations and the asymptotic structure of corresponding radiative spacetimes has been achieved, we have seen that open problems remain. Curiously enough, in the case of vacuum Einstein's equations with a *non-vanishing cosmological constant* a more complete picture is known for some time already. By using his regular conformal field equations, Friedrich demonstrated [6] that initial data sufficiently close to de-Sitter data develop into solutions of Einstein's equations with a positive cosmological constant, which are asymptotically simple (with a smooth conformal infinity), as required in the original framework of Penrose. Later Friedrich [7] also discussed the existence of asymptotically simple solutions to the Einstein vacuum equations with a negative cosmological constant.

In his more recent investigations [3], Friedrich constructed the new - finite but "wider" than the point  $i_0$  - representation of spacelike infinity. This construction enables one to make much deeper analysis of the initial data in the region where null infinity touches spacelike infinity. Good chances now exist to obtain clear criteria determining which data lead to the smooth and which just to the polyhomogeneous null infinity.

The ultimate goal of rigorous work on the existence and asymptotics of solutions of the Einstein equations is *physics*: one hopes to be able to consider astrophysical sources, to relate their behaviour to the characteristics of the far fields. One would like to have under control various (both analytical and

numerical) approximation procedures. A still more ambitious program is to consider strong initial data so as to be able to analyze such issues as cosmic censorship.

In the following we shall briefly discuss three classes of *explicit* radiative solutions of Einstein's equations: cylindrical waves, Robinson-Trautman and type N spacetimes, and the boost-rotation symmetric spacetimes. The latter represent the only known examples describing moving, radiating objects; except for points of null infinity where particles start and end, the null infinity is smooth. We here closely follow our recent, more detailed reviews [8,9] in which also other classes of radiative spacetimes are analyzed, such as plane waves and gravitational waves representing inhomogeneous cosmological models.

### 3 Cylindrical waves

Despite the fact that cylindrically symmetric waves cannot describe exactly the radiation from bounded sources, they even recently played an important role in clarifying a number of complicated issues, such as testing the quasilocal mass-energy, testing codes in numerical relativity, investigation of the cosmic censorship, and quantum gravity.

In work with Ashtekar and Schmidt [10,11], we considered gravitational waves with a space-translation Killing field (“generalized Einstein-Rosen waves”). In (2+1)-dimensional framework the Einstein-Rosen subclass forms a simple instructive example of explicitly given spacetimes which admit a smooth global null (and timelike) infinity even for strong initial data.

4-dimensional vacuum gravity which admits a spacelike hypersurface Killing vector  $\partial/\partial z$  is equivalent to 3-dimensional gravity coupled to a scalar field. In 3 dimensions, there is no gravitational radiation. Hence, the local degrees of freedom are all contained in the scalar field. One therefore expects that Cauchy data for the scalar field will suffice to determine the solution. For data which fall off appropriately, we thus expect the 3-dimensional Lorentzian geometry to be asymptotically flat in the sense of Penrose, i.e. that there should exist a 2-dimensional boundary representing null infinity. In general cases, this is analyzed in [10].

Restricting ourselves to the Einstein-Rosen waves by assuming that there is a further spacelike, hypersurface orthogonal Killing vector  $\partial/\partial\varphi$  which commutes with  $\partial/\partial z$ , we find the 3-metric given by

$$d\sigma^2 = g_{ab}dx^a dx^b = e^{2\gamma}(-dt^2 + d\rho^2) + \rho^2 d\varphi^2. \quad (2)$$

The field equations become

$$-\ddot{\psi} + \psi'' + \rho^{-1}\psi' = 0, \quad \gamma' = \rho(\dot{\psi}^2 + \psi'^2), \quad \dot{\gamma} = 2\rho\dot{\psi}\psi'. \quad (3)$$

Thus, we can first solve the axisymmetric wave equation for  $\psi$  on Minkowski space and then solve for  $\gamma$  – the only unknown metric coefficient – by quadratures.

By analyzing the asymptotic behavior of the solutions we can conclude that *cylindrical waves in (2+1)-dimensions give an explicit model of the Bondi-Penrose radiation theory which admits smooth null and timelike infinity for arbitrarily strong initial data*. There is no other such model available. The general results on the existence of  $\mathcal{I}$  in 4 dimensions assume weak data.

## 4 On the Robinson-Trautman and type N twisting solutions

These spacetimes have attracted increased attention in the last decade – most notably in the work by Chruściel, and Chruściel and Singleton [12]. In these studies the Robinson-Trautman spacetimes have been shown to exist globally for all positive “times”, and to converge asymptotically to a Schwarzschild metric. Interestingly, the extension of these spacetimes across the “Schwarzschild-like” event horizon can only be made with a finite degree of smoothness. These studies are based on the derivation and analysis of an asymptotic expansion describing the long-time behaviour of the solutions of the nonlinear parabolic Robinson-Trautman equation.

In our work [13,14] we studied Robinson-Trautman spacetimes with a positive cosmological constant  $\Lambda$ . The results proving the global existence and convergence of the solutions of the Robinson-Trautman equation can be taken over from the previous studies since  $\Lambda$  does not explicitly enter this equation. We have shown that, starting with arbitrary, smooth initial data at  $u = u_0$ , these cosmological Robinson-Trautman solutions converge exponentially fast to a Schwarzschild-de Sitter solution at large retarded times ( $u \rightarrow \infty$ ). The interior of a Schwarzschild-de Sitter black hole can be joined to an “external” cosmological Robinson-Trautman spacetime across the horizon  $\mathcal{H}^+$  with a higher degree of smoothness than in the corresponding case with  $\Lambda = 0$ . In particular, in the extreme case with  $9\Lambda m^2 = 1$ , in which the black hole and cosmological horizons coincide, the Robinson-Trautman spacetimes can be extended smoothly through  $\mathcal{H}^+$  to the extreme Schwarzschild-de Sitter spacetime with the same values of  $\Lambda$  and  $m$ . However, such an extension is not analytic (and not unique).

We have also demonstrated that the cosmological Robinson-Trautman solutions represent explicit models exhibiting the cosmic no-hair conjecture. As far as we are aware, these models represent the only exact analytic demonstration of the cosmic no-hair conjecture under the presence of gravitational waves. They also appear to be the only exact examples of black hole formation in nonspherical spacetimes which are not asymptotically flat.

### Type N twisting spacetimes

Since diverging, non-twisting Robinson-Trautman spacetimes of type N have singularities, there has been hope that if one admits a nonvanishing twist a more realistic radiative spacetime may exist.

Stephani [15], however, indicated, by constructing a general solution of the linearized equations, that singularities at infinity probably exist. More recently, Finley et al [16] found an approximative twisting type N solution up to the third order of iteration on the basis of which they suggested that it seems that the twisting, type N fields can describe a radiation field outside bounded sources. However, employing the Newman-Penrose formalism and MAPLE we succeeded in discovering a nonvanishing quartic invariant in the 2nd derivatives of the Riemann tensor [17], which shows that solutions of both Stephani and Finley et al contain singularities at large  $r$ . Mac Alevey [18] argued that an approximate solution at any finite order can be calculated without occurrence of singularities. It is very likely, however, that a corresponding exact solution must contain singularities since Mason [19] proved that the only vacuum algebraically special spacetime that is asymptotically simple is the Minkowski space.

Even if a radiative solution with complete smooth null infinity may be out of reach, it is of interest to construct radiative solutions which at least admit a global null infinity in the sense that its smooth cross sections exist although this null infinity is not necessarily complete. The only explicit examples of such solutions are spacetimes with boost-rotation symmetry.

## 5 The boost-rotation symmetric radiative spacetimes

I reviewed these spacetimes representing “uniformly accelerated objects” (see Fig. 2) in various places (see e.g. [8,20] and references therein); here I shall just mention some new results. The Penrose diagram of these spacetimes is schematically illustrated in Fig. 3.

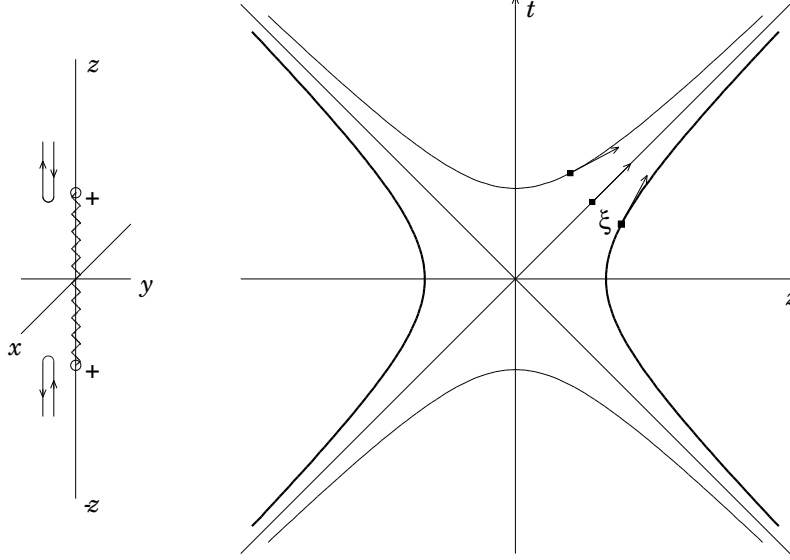
The unique role of the boost-rotation symmetric spacetimes is exhibited by a theorem [21] which roughly states that in axially symmetric, locally asymptotically flat electrovacuum spacetimes (in the sense that a null infinity satisfying Penrose’s requirements exists, but it need not necessarily exist globally), the only additional symmetry that does not exclude radiation is the *boost* symmetry.

To prove such result we start from the metric

$$\begin{aligned}
 ds^2 = & (r^{-1} V e^{2\beta} - r^2 e^{2\gamma} U^2 \cosh 2\delta - r^2 e^{-2\gamma} W^2 \cosh 2\delta - 2r^2 U W \sinh 2\delta) du^2 \\
 & + 2e^{2\beta} du dr + 2r^2 (e^{2\gamma} U \cosh 2\delta + W \sinh 2\delta) du d\theta \\
 & + 2r^2 (e^{-2\gamma} W \cosh 2\delta + U \sinh 2\delta) \sin \theta du d\phi \\
 & - r^2 [\cosh 2\delta (e^{2\gamma} d\theta^2 + e^{-2\gamma} \sin^2 \theta d\phi^2) + 2 \sinh 2\delta \sin \theta d\theta d\phi] ,
 \end{aligned} \tag{4}$$

where all functions describing metric and electromagnetic field tensor  $F_{\mu\nu}$  are independent of  $\phi$ . Assuming asymptotic expansions of these functions at large  $r$  with  $u, \theta, \phi$  fixed to guarantee asymptotic flatness, and using the outgoing





**Fig. 2.** Two particles uniformly accelerated in opposite directions.

radiation condition and the field equations, one finds these expansions to have specific forms. For example,

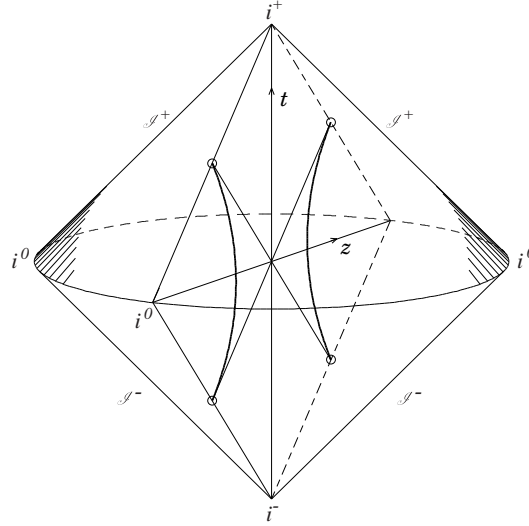
$$\begin{aligned} \gamma &= \frac{c}{r} + (C - \tfrac{1}{6}c^3 - \tfrac{3}{2}cd^2)\frac{1}{r^3} + \dots, & V &= r - 2M + \dots, \\ F_{02} &= X + (\epsilon_{,\theta} - e_{,u})\frac{1}{r} + \dots, & F_{03} &= Y - \frac{f_{,u}}{r} + \dots, \end{aligned} \quad (5)$$

where the ‘coefficients’  $c, d, \dots$  are functions of  $u$  and  $\theta$ . The expansions are needed to further orders – see [21] for their forms. Let us only recall that the decrease of the Bondi mass,  $m(u) = \frac{1}{2} \int_0^\pi M(u, \theta) \sin \theta d\theta$ , is given by

$$m_{,u} = -\tfrac{1}{2} \int_0^\pi (c_{,u}^2 + d_{,u}^2 + X^2 + Y^2) \sin \theta d\theta \leq 0, \quad (6)$$

where  $c_{,u}, d_{,u}, X, Y$  are gravitational and electromagnetic news functions.

Now one writes down the Killing equations and solves them asymptotically in  $r^{-1}$ . One arrives at the following theorem [21]: Suppose that an axially symmetric electrovacuum spacetime admits a “piece” of  $\mathcal{J}^+$  in the sense



**Fig. 3.** The Penrose compactified diagram of a boost-rotation symmetric spacetime. Null infinity can admit smooth sections.

that the Bondi-Sachs coordinates can be introduced in which the metric takes the form (4), with the asymptotic form of the metric and electromagnetic field given by (5). If this spacetime admits an additional Killing vector forming with the axial Killing vector a two-dimensional Lie algebra then the additional Killing vector has asymptotically the form

$$\eta^\alpha = [-ku \cos \theta + \alpha(\theta), kr \cos \theta + \mathcal{O}(r^0), -k \sin \theta + \mathcal{O}(r^{-1}), \mathcal{O}(r^{-1})], \quad (7)$$

where  $k$  is a constant. For  $k = 0$  it generates asymptotically translations (function  $\alpha$  has then a specific form). For  $k \neq 0$  it is the boost Killing field.

The case of translations is analyzed in detail in [22]. Theorem 1, precisely formulated and proved there, states that if asymptotically translational Killing vector is spacelike, then null infinity is singular at some  $\theta \neq 0, \pi$ ; if it is null, null infinity is singular at  $\theta = 0$  or  $\pi$ . The first case corresponds to cylindrical waves, the second case to a plane wave propagating along the symmetry axis. We refer to [22] for the case when there is also a cosmic string present along the symmetry axis. The case of timelike Killing vector is described by Theorem 2 (proved also in [22]): “If an axisymmetric electrovacuum spacetime with a non-vanishing Bondi mass admits an asymptotically translational Killing vector and a complete cross section of  $\mathcal{J}^+$ , then the translational Killing vector is timelike and spacetime is thus stationary.”

The case of the boost Killing vector ( $k \neq 0$ ) is thoroughly analyzed in [21]. The general functional forms of the news functions (both gravitational and electromagnetic), and of the mass aspect and total Bondi mass of boost-rotation symmetric spacetimes are there given. Recently these results were obtained [23] by using the Newman-Penrose formalism and under more general assumptions (for example,  $\mathcal{J}$  could in principle be polyhomogeneous).

The general structure of the boost-rotation symmetric spacetimes with hypersurface orthogonal Killing vector was analyzed in detail in [24]. Their radiative properties, including explicit construction of radiation patterns and of Bondi mass for the specific boost-rotation symmetric solutions were investigated in several works – we refer to the reviews [8,20] and [25] for details. There also the role of the boost-rotation symmetric spacetimes in such diverse fields like numerical relativity and quantum production of black-hole pairs is noticed and references are given.

Here I would like to mention yet a recent progress in understanding specific boost-rotation symmetric spacetimes with Killing vectors which are *not* hypersurface orthogonal. This is the *spinning C*-metric (see e.g. [26]). It was discovered by Plebański and Demiański as a generalization of the standard *C*-metric which is known to represent uniformly accelerated non-rotating black holes. In [27] we first transformed the metric into Weyl coordinates, and then found a transformation which brings it into the canonical form of the radiative spacetimes with the boost-rotation symmetry:

$$\begin{aligned}
 ds^2 = & e^\lambda d\rho^2 + \rho^2 e^{-\mu} d\phi^2 \\
 & + (z^2 - t^2)^{-1} [(e^\lambda z^2 - e^\mu t^2) dz^2 - 2zt(e^\lambda - e^\mu) dz dt \\
 & + (e^\lambda t^2 - e^\mu z^2) dt^2] - 2\mathcal{A}e^\mu(zdt - tdz)d\phi - \mathcal{A}^2 e^\mu(z^2 - t^2)d\phi^2, \quad (8)
 \end{aligned}$$

where functions  $e^\mu, e^\lambda$  and  $\mathcal{A}$  are given in terms of  $(t, \rho, z)$  in a somewhat complicated but explicit manner. This metric can represent two uniformly accelerated, spinning black holes, either connected by a conical singularity, or with conical singularities extending from each of them to infinity. The behaviour of curvature invariants clearly indicates the presence of a non-vanishing radiation field (see Figure 5 in [27]). The spinning *C*-metric is the only explicitly known example with two Killing vectors which are not hypersurface orthogonal, in which one can give arbitrarily strong initial data on hyperboloid “above the roof” ( $t > |z|$ ) which evolve into the radiative spacetime with smooth  $\mathcal{J}^+$ .

## References

1. Bondi, H., van der Burg, M. G. J. and Metzner, A. W. K. (1962) *Gravitational Waves in General Relativity. VII. Waves from Axis-symmetric Isolated Systems*, Proc. Roy. Soc. Lond. A **269**, 21

2. Penrose R. (1963) *Asymptotic properties of fields and space-times*, Phys. Rev. Lett. **10**, 66. How this framework became powerful is especially evident from the 2-volume monograph by R. Penrose and W. Rindler, *Spinors & space-time*, Cambridge University Press, Cambridge 1986.
3. Friedrich, H. (1998) *Einstein's Equation and Geometric Asymptotics*, in *Gravitation and Relativity: At the turn of the Millenium* (Proceedings of the GR-15 conference), eds. N. Dadhich and J. Narlikar, Inter-University Centre for Astronomy and Astrophysics Press, Pune
4. Christodoulou D. and Klainerman S., *The nonlinear stability of the Minkowski spacetime*, Princeton University Press, Princeton 1994.
5. Chruściel, P., MacCallum, M. A. H. and Singleton, P. B. (1995) *Gravitational waves in general relativity XIV. Bondi expansions and the 'polyhomogeneity' of  $\mathcal{I}$* , Phil. Trans. Roy. Soc. Lond. **A350**, 113
6. Friedrich, H. (1986) *On the existence of  $n$ -geodesically complete or future complete solutions of Einstein's field equations with smooth asymptotic structure*, Commun. Math. Phys. **107**, 587
7. Friedrich, H. (1995) *Einstein equations and conformal structure: existence of anti-de Sitter-type space-times*, J. Geom. Phys. **17**, 125
8. Bičák, J. (2000) *Selected solutions of Einstein's field equations: their role in general relativity and astrophysics*, in *Einstein's Field Equations and Their Physical Meaning*, ed. B. G. Schmidt, Springer Verlag
9. Bičák, J. (2000) *Exact radiative spacetimes: some recent developments*, Annalen Phys. **9**, 207-216
10. Ashtekar, A., Bičák, J. and Schmidt, B. G. (1997) *Asymptotic structure of symmetry-reduced general relativity*, Phys. Rev. **D55**, 669
11. Ashtekar, A., Bičák, J. and Schmidt, B. G. (1997) *Behaviour of Einstein-Rosen waves at null infinity*, Phys. Rev. **D55**, 687
12. Chruściel, P. T. (1992) *On the global structure of Robinson-Trautman space-times*, Proc. Roy. Soc. Lond. A **436**, 299; Chruściel, P. T., Singleton, D. B. (1992) *Non-Smoothness of Event Horizons of Robinson-Trautman Black Holes*, Commun. Math. Phys. **147**, 137, and references therein
13. Bičák, J., Podolský, J. (1997) *The global structure of Robinson-Trautman radiative space-times with cosmological constant*, Phys. Rev. **D55**, 1985
14. Bičák, J., Podolský, J. (1995) *Cosmic no-hair conjecture and black-hole formation: An exact model with gravitational radiation*, Phys. Rev. **D52**, 887
15. Stephani, H. (1993) Class. Quantum Grav. **10**, 2187
16. Finley, J. D., Plebański, J. F. and Przanowski, M. (1997) Class. Quant. Grav. **14**, 487
17. Bičák, J., Pravda, V. (1998) *Curvature invariants in type N spacetimes*, Class. Quantum Grav. **15**, 1539
18. MacAlevey, P. (1999) Class. Quantum Grav. **16**, 2259
19. Mason, L. (1998) Class. Quantum Grav. **15**, 1019
20. Bičák, J. (1997) *Radiative spacetimes: Exact approaches*, in *Relativistic Gravitation and Gravitational Radiation* (Proceedings of the Les Houches School of Physics), eds. J.-A. Marck and J.-P. Lasota, Cambridge University Press, Cambridge
21. Bičák, J., Pravdová, A. (1998) *Symmetries of asymptotically flat electrovacuum spacetimes and radiation*, J. Math. Phys. **39**, 6011
22. Bičák, J., Pravdová, A. (1999) *Axisymmetric electrovacuum spacetimes with a translational Killing vector at null infinity*, Class. Quantum Grav. **16**, 2023

- 23. Valiente-Kroon, J. A. (1999) J. Math. Phys., to appear
- 24. Bičák, J., Schmidt, B. G. (1989) *Asymptotically flat radiative space-times with boost-rotation symmetry: the general structure*, Phys. Rev. **D40**, 1827
- 25. Pravda, V., Pravdová, A. (2000) Czech. J. Physics, to appear
- 26. Kramer, D., Stephani, H., Herlt, E. and MacCallum, M. A. H. (1980) *Exact solutions of Einstein's field equations*, Cambridge University Press, Cambridge
- 27. Bičák, J., Pravda, V. (1999) *Spinning C-metric: radiative spacetime with accelerating, rotating black holes*, Phys. Rev. **D60**, 044004